Mathematics 131B, Fall 2016 – ℝami Luisto
Homework 8 - Due date Friday November 25th.

You are welcomed, even encouraged to discuss the homework problems together. If and when you do you should, however, acknowledge your collaboration and mention who you have been working with in your returned solutions. I sincerely hope that it goes without saying that copying someone else's work without thought is not 'working together', and while not technically disallowed, is very counterproductive for your learning process. While in certain previous courses one might get a good grade just my memorizing a few solution algorithms, many problems in this course require some level of understanding what is going on mathematically. This will be the rule rather than the exception in all further courses, and trying to mimic proofs without thinking will lead to misery after the first few weeks. Understanding what is happening, on the other hand, will lead to feelings of **great beauty** and **elegance**.

If you are having problems with an exercise, you can and should ask for help. However, if you glance at a problem and immediately shout "I don't know how to solve this!", you are not having problems with the exercise but rather you are having problems with not expending any effort to solve the problem. **"You never call any question impossible, said Harry, until you have taken an actual clock and thought about it for five minutes, by the motion of the minute hand. Not five minutes metaphorically, five minutes by a physical clock."** (*Harry Potter in Harry Potter and the Methods of Rationality.*) `\end{rant}`

Prepping problems. These problems will not be graded. They are voluntary, very basic problems that can help you get up to speed.

p1) Show that $\cos(0) = 1$ and $\sin(0) = 0$ by using the definition via the complex exponential map.
p2) Show that the map $x \mapsto \exp(ix)$ is 2π periodic.
p3)

Homework problems

(1) Show that $\cos(-x) = \cos(x)$ and $\sin(-x) = -\sin(x)$ by using the definition via the complex exponential map.
(2) Show that $\sin'(x) = \cos(x)$,
  (a) by using the definition via the complex exponential map. (Here you may use the fact that also for complex-valued differentiable maps $D(f \circ g)(z) = g'(z)f'(g(z))$.)
  (b) by using the expression of the sine and cosine as formal power series.
(3) Show that a periodic continuous function in $C(\mathbb{R}, \mathbb{R})$ is always bounded.
(4) Let $f \in C(\mathbb{R}/\mathbb{Z}, \mathbb{R})$ be a constant function, i.e. $f(x) = c$ for all $x \in \mathbb{R}$. Calculate $\hat{f}(n)$ for all $n \in \mathbb{Z}$.
  What is the Fourier series of a constant function?

The following is an extra problem to those who want to learn something more. It will not be graded.

Extra challenge: Suppose for a moment that the dirac delta function exists, i.e. we have a function $\delta\colon [0,1) \to \mathbb{R}$ with the property that

$$\int_{[0,1)} \delta(s)f(s)ds = f(0)$$

for any continuous function f.

What would be the Fourier series of such a function?

Note that since the Dirac delta is supposed to be some sort of limit of approximations of identity, in a perfect world the partial sums of the Fourier series of δ should look more and more like approximations to the identity. Study the partial sums behind the following link and see if this is the case. `https://www.desmos.com/calculator/sdo7zuttly`

On the other hand, for any $N \in \mathbb{N}$ we can study the mapping

$$\delta_N(x) = \sum_{j=-N}^{N} \hat{\delta}(j)e_j(x)$$

with $e_j(x) = \mathrm{e}^{-2\pi ijx}$. Show that if $f = \sum_{j=-\infty}^{\infty} \hat{f}(j)e_j$, then

$$\lim_{N\to\infty} \int_{[0,1)} \delta_N(s)f(s)\,ds = f(0).$$