Mathematics 131B, Fall 2016 – ℝami Luisto
Midterm I on October 17:th.

Name (use a pen):

Student ID (use a pen):

Signature (use a pen):

Rules:

- Duration of the exam: **50 minutes.**
- By writing your name and signature on this exam paper, you attest that you are the person indicated and will adhere to the UCLA Student Conduct Code.
- No calculators, computers, cell phones (all the cell phones should be turned off during the exam), notes, books or other outside material are permitted on this exam. If you need more scratch paper, you should ask for it from one of the supervisors. Do not use your own scratch paper!
- Please justify all your answers with mathematical precision and write rigorous and clear proofs. You may loose points in the lack of justification of your answers.
- If using results proven in the book or lectures, state the theorem you are using.
- **This Midterm has 5 problems from which you should choose 4 to solve.** Each problem is worth six (6) points.
- I wish you success!

Problem	Score
Question 1	
Question 2	
Question 3	
Question 4	
Question 5	
Total	

Questions: Choose four (4) of the following five questions!

Q1. We define a map $d_\infty\colon \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ by setting

$$d_\infty(\mathbf{x}, \mathbf{y}) = \max\{|x_1 - y_1|, \dots, |x_n - y_n|\}.$$

Show that d_∞ is a metric in $\mathbb{R}^n$.

Q2. (a) (4p) Let (X, d) be a metric space. Let be $U_1, \dots, U_k$ open subsets of X. Show that the intersection $\bigcap_{j=1}^{k} U_j$ is open in X.

(b) (2p) Give an examle of open sets V_i, $i \in \mathbb{N}$ in the metric space $(\mathbb{R}^2, d_e)$ whose intersection $\bigcap_{j\in\mathbb{N}} V_j$ is not open.

Q3. Let (X, d) be a metric space.

(a) (2p) Show that if a sequence (x_n) converges in X, then it is a Cauchy sequence.

(b) (4p) Show that if X is compact, then it is complete.

Q4. Equip $\mathbb{R}^2$ with the following metric:

$$d_1(\mathbf{x}, \mathbf{y}) = |x_1 - y_1| + |x_2 - y_2|.$$

(You may assume that d_1 is a metric.)

(a) (2p) Show that the mapping $p\colon \mathbb{R}^2 \to \mathbb{R}$, $p(x_1, x_2) = x_1 + x_2$ is continuous.

(b) (2p) Show that the set

$$U = \{(x, y) \in \mathbb{R}^2 \mid x^2 + xy - 4y^2x > 5\}$$

is open in $\mathbb{R}^2$. (Here you may assume that multivariable polynomials are continuous with respect to the d_1-metric and use any theorems proven in the course.)

(c) (2p) Set $V = \mathbb{R}^2 \setminus \{(0, 0)\}$. What is the boundary ∂V of V?

Q5. Take $C([0, 1], \mathbb{R})$ be the space of all continuous functions $f\colon [0, 1] \to \mathbb{R}$, and set $d_\infty\colon C([0, 1], \mathbb{R}) \times C([0, 1], \mathbb{R}) \to \mathbb{R}$ to be the metric

$$d_\infty(f, g) = \sup_{x\in[0,1]} |f(x) - f(y)|.$$

You may assume that d_∞ is a metric in $C([0, 1], \mathbb{R})$.

Define a mapping $I\colon C([0, 1], \mathbb{R}) \to \mathbb{R}$ by $I(f) = f(1) - f(0)$.

(a) (4p) Show that

$$|I(f) - I(g)| \le 2d_\infty(f, g)$$

for all $f, g \in C([0, 1], \mathbb{R})$.

(b) (2p) Show that I is continuous.