Mathematics 131B, Fall 2016 – $\mathbb{R}$ami Luisto
Midterm II on November 7th.

Name (use a pen):

Student ID (use a pen):

Signature (use a pen):

Rules:

- Duration of the exam: **50 minutes.**
- By writing your name and signature on this exam paper, you attest that you are the person indicated and will adhere to the UCLA Student Conduct Code.
- No calculators, computers, cell phones (all the cell phones should be turned off during the exam), notes, books or other outside material are permitted on this exam. If you need more scratch paper, you should ask for it from one of the supervisors. Do not use your own scratch paper!
- Please justify all your answers with mathematical precision and write rigorous and clear proofs. You may lose points in the lack of justification of your answers.
- If using results proven in the book or lectures, state the theorem you are using.
- **This Midterm has 4 problems from which you should choose 3 to solve.** Each problem is worth six (6) points.
- I wish you success!

Problem	Score
Question 1	
Question 2	
Question 3	
Question 4	
Total	

Questions: Choose three (3) of the following four questions!

Q1. (a) (2p) Define the concepts of pointwise convergence and uniform convergence of a sequence of functions $\mathbb{R} \to \mathbb{R}$.
(b) (4p) Show that the sequence $f_n \colon \mathbb{R} \to \mathbb{R}$, $f_n(x) = \frac{x}{n}$ converges pointwise but not uniformly.

Q2. (a) (4p) Suppose f_n is a sequence of continuous functions $[0,1] \to \mathbb{R}$ such that

$$\sum_{n=1}^{\infty} \|f_n\|_\infty < \infty.$$

Show that the sequence $S_k = \sum_{j=1}^{k} f_j$ is a Cauchy sequence with respect to the metric d_∞ of $C([0,1],\mathbb{R})$.
(You may use results from previous courses concerning converging series. Do not, however, directly use the Weierstrass M-test.)
(b) (2p) Define $f_n \colon \mathbb{R} \to \mathbb{R}$ by setting $f_n(x) = \frac{1}{2^n}\cos(nx^2)$. Show that $\sum_{n=1}^{\infty} f_n$ converges uniformly to a function $\mathbb{R} \to \mathbb{R}$.
(You may use any results from previous courses or our course.)

Q3. (a) (2p) What is the definition of a function $f \colon \mathbb{R} \to \mathbb{R}$ having compact support?
(b) (4p) Let $f, g \colon \mathbb{R} \to \mathbb{R}$ be two continuous mappings with compact support. Show that their convolution $f * g$ also has compact support.

Q4. Let $f(x) = \sum_{k=1}^{\infty} a_k x^k$ be a formal power series. Denote the radius of convergence as

$$R = \frac{1}{\limsup_{n\to\infty} |a_n|^{1/n}}.$$

Show that the formal power series converges pointwise $(-R, R)$.

(In this problem you may assume that a geometric series $\sum_{k=1}^{\infty} c^k$, $c \in (0,1)$, converges. Alternatively you may use the root test for series from previous courses if you remember it.)